Efficiency of the Stirling Engine Cycle

Candidate Number: 265095
November 2023

***Abstract*—The objective on this experiment was to obtain the practical and theoretical efficiency for a Stirling Engine and to demonstrate a linear relationship between flywheel frequency and input power. The practical efficiency was calculated using a filament of known input power. A theoretical efficiency of 0.161 ± 0.0007 was obtained by analyzing isotherms on a PV graph, with an input power of (33.8 ± 0.2)W.**

TABLE I
PRACTICAL EFFICIENCY DATA

Peak Voltage	Input Power	Output Power	Practical Efficiency
V_p (V)	P_{in} (W)	P_{out} (W)	η_P
8.4 ± 0.05	33.8 ± 0.2	0.687 ± 0.004	$(2.0 \pm 0.02) \times 10^{-2}$
10.4 ± 0.05	51.6 ± 0.2	1.126 ± 0.01	$(2.2 \pm 0.02) \times 10^{-2}$
12.4 ± 0.05	74.5 ± 0.3	2.180 ± 0.001	$(2.9 \pm 0.01) \times 10^{-2}$

I. INTRODUCTION

A Stirling Engine is a simple engine that exploits a temperature differential to generate useful work. The size of this temperature differential determines the theoretical efficiency of the engine. The practical efficiency is less, determined also by other factors, such as the material properties of the engine components or friction.

The Stirling Engine investigated used a filament connected to an AC power supply as the hot reservoir. The cold reservoir was provided by a constant supply of cold water in a jacket around the piston cylinder. The piston has a small hole containing copper wool, acting as a *regenerator*.[1] The piston, pushed by the hot expanding air, drives a flywheel, which falls, driving the piston up to complete the cycle.

II. THEORY

The practical efficiency is the ratio of work into and useful work out of a system. To calculate this, a means to measure both input power P_{in} and output power P_{out} was needed. The equation for Joule heating, $P = IV$ is not valid for AC current.[2] Instead, P_{in} must use the root-mean square values of current and voltage when divided by a factor of $\sqrt{2}$. Such that

$$P_{in} = \frac{I}{\sqrt{2}}\frac{V_p}{\sqrt{2}} = \frac{1}{2}IV_p. \tag{1}$$

Where I is current (A) and V_p is peak voltage (V). To calculate the output power P_{out}, the work done per cycle is needed. Making the assumption that the entropy of the air within the engine does not change over a cycle. Plotting a graph of P versus V over one cycle will show two isotherms corresponding to the engine motion. The area encircled by these isotherms is equal to the work done, W, per cycle. Confirmation by dimensional analysis;

$$W = Fs \Rightarrow \frac{LM}{T^2}L = \frac{LM}{L^2T^2}L^3 \Rightarrow \frac{F}{A}V = PV \tag{2}$$

The angular frequency of the flywheel, ω, must also be found in order to calculate P_{out}.

$$P_{out} = \frac{W\omega}{2\pi} \tag{3}$$

It follows then, that the practical efficiency of the engine is the ratio of these two values,

$$\eta_P = \frac{P_{out}}{P_{in}}. \tag{4}$$

A plot of P_{in} versus ω should have a linear relationship, assuming the work done and practical efficiency of the engine do not change significantly, shown by rearranging (4),

$$\omega = \frac{W\eta_P}{2\pi}P_{in}. \tag{5}$$

Calculating the theoretical efficiency, η_T, using the PV graph is possible, due to the ideal gas law,

$$PV = nRT. \tag{6}$$

Where n is the number of mols of gas, $R \approx 8.3144$, is the molar gas constant. T is the temperature of the gas. By using a measured temperature, T_C, for the cold isotherm, the number of moles inside the piston cylinder can be determined. This can be used to calculate the temperature, T_H, of the hot isotherm. The theoretical efficiency is then given as

$$\eta_T = 1 - \frac{T_c}{T_h}. \tag{7}$$

III. METHOD

The CASSY Lab 2 Measurement System software, connected to a rotary motion sensor (Model LD Didactic 524 082) was used to measure pressure and volume. To calibrate the software, we set two variables. $S_0 = (12.03 \pm 0.005)$cm and $S_{A1} = (7.41 \pm 0.005)$cm. The following formula was inputted into the software,

$$V = 28.3(S_0 - S_{A1}) + 195. \tag{8}$$

With the volume, V, in cm^3. The software was setup to produce a PV graph over one cycle, measuring both every 10ms. To measure the frequency of the flywheel, a small piece of protruding paper was attached. A digital light gate was then positioned to record the paper breaking the beam of light, from which angular frequency was measured.

After the water was switched on for the cold reservoir, the voltage was adjusted to 10V initially, giving a reading of

(10.4 ± 0.05)V on the multimeter. The current on the ammeter was recorded as (9.93 ± 0.005)A.

To start the engine, the flywheel was rotated to facilitate reaching a self-sustaining pace, this is why 10V was chosen to start. The lightgate was then programmed five times to record counts for a thirty second period.

The CASSY software produced a PV graph for each cycle, the software was used ten times to calculate the area inside the graph, equal to W. This procedure was repeated for the other two voltages, 8V and 12V.

Once the measurements had been made, the voltage was switched back to 8V. The CASSY software was used to download a CSV file of the PV graph. The middle section was isolated between 240cm^3 and 280cm^3 as two isotherms for analysis. The cold water was measured to be $T_C = (284 \pm 0.25)$K using a thermometer. Number of mols, n, was calculated as $(1.87 \pm 0.005) \times 10^{-2}$mol. These values were used to calculate the hot isotherm temperature, $T_H = (338.65 \pm 0.03)K$.

IV. Results and Discussion

A Jupyter Notebook using Python was used to manipulate the data. Errors were calculated using the variance of each set of results. The results show that increasing the input power results in increased practical efficiency. A higher input power makes for a higher filament temperature. This follows from equation (7) that increasing the temperature difference results in a more efficient Stirling engine.

TABLE II
Practical Efficiency Data

Peak Voltage V_p (V)	Input Power P_{in} (W)	Output Power P_{out} (W)	Practical Efficiency η_P
8.4 ± 0.05	33.8 ± 0.2	0.687 ± 0.004	$(2.0 \pm 0.02) \times 10^{-2}$
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A plot of ω versus P_{in} with a least squares linear fit gives a gradient of $m = 0.06 \pm 0.001$ and offset $c = 0.51 \pm 0.07$. The reduced χ^2 of this fit is 1.81. The cause of the discrepancy is that not enough time was given for the engine to reach its operating equilibrium during the initial measurement at 10.4V. The standard deviation of this set of work done data is eight times that of the other two sets.

The chosen range of values for the isotherms was a good fit for a linear approximation. The graph of residuals mostly fall between one and negative one. The graph gives reduced χ^2 values of 0.21 and 0.06 for T_C and T_H respectively.

The final result for theoretical efficiency, obtained from analyzing the isotherms, $\eta_T = 0.161 \pm 0.0007$, seems lower than expected. Together with the low value of $T_H = (65.65 \pm 0.03)C^o$. This is much lower than the 1000K+ temperature of the filament. Rather this value corresponds to the air temperature inside the working chamber. Air is not a very good conductor and won't increase much in temperature over one cycle.

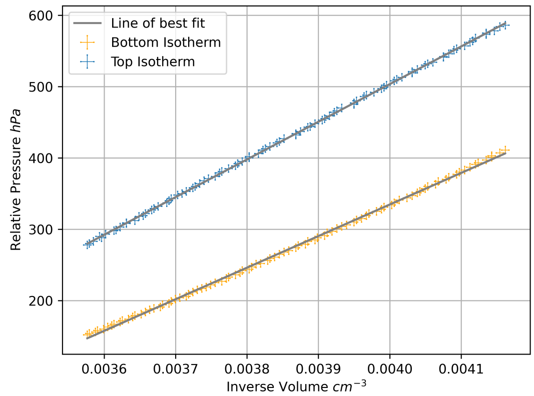

Fig. 1. Graph of relative pressure (hPa) versus inverse volume (cm^3). The two isotherms cut from the PV graph have both had a linear fit applied.

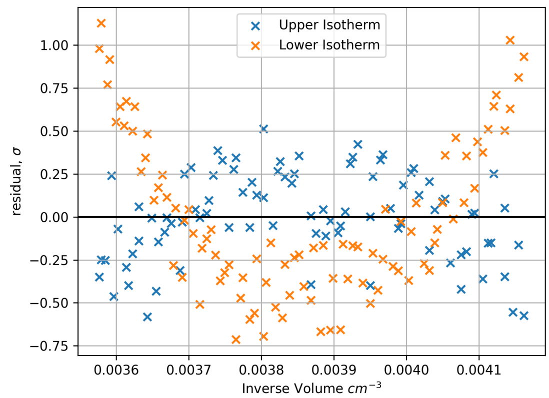

Fig. 2. Graph of residuals for both linear fits. The fit is generally good as most absolute values are less than one. The spread, however, reflects that this is part of a larger curve.

V. Conclusion

A Stirling Engine driven by a filament can be used to calculate both its practical efficiency and theoretical efficiency. This engine having values of $\eta_P \approx 0.025$ and $\eta_T = 0.161$. A linear relationship was also demonstrated between input power and flywheel frequency with a reduced $\chi^2 = 1.81$. Letting the engine reach equilibrium would have reduced this value. One other improvement would be to obtain values of η_T for the other two voltages for comparison and consistency. The low measurement of $T_H = (65.65 \pm 0.03)C^o$ implies that a gas with a higher thermal conductivity than air would increase η_T. Other literature[13] mention using high thermal conductivity gases, Hydrogen and Helium, in Stirling Engines.

References

[1] Appendix C: The Stirling Engine, Integration of Alternative Sources of Energy, Felix A. Farret and M. Godoy Sim oes, 2006 John Wiley and Sons, Inc.
[2] Boston University Website Page on Joule Heating,https://physics.bu.edu/py106/notes/Power.html, last accessed 11/11/2023
[3] Chapter 19, Stirling Cycle Engines: Inner Workings and Design, Allan J Organ, 2014 John Wiley and Sons, Ltd.